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MULTIPLE-CRITERIA DECISION-MAKING BASED ON THE USE OF SINGLE-VALUED NEUTROSOPHIC SETS AND SIMILARITY MEASURES

Abstract. As the generalization of the fuzzy and similar sets based on Fuzzy sets, neutrosophic sets provide significant possibilities in the case of solving complex decision problems, often related to uncertainty and unreliability. Neutrosophic sets use three values named the truth degree, the indeterminacy degree and the falsity degree, which allow for a more accurate evaluation of alternatives in relation to complex evaluation criteria. As a result of their application in solving numerous different decision-making problems, several approaches to their ranking have been proposed. Therefore, this paper provides a comprehensive overview of the approaches to the ranking of single-valued neutrosophic numbers and a comparison of the results obtained by using them. Finally, numerical illustrations are given.
Keywords: neutrosophy, single-valued neutrosophic set, similarity measures, MCDM.

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1. Introduction

In recent decades, multi-criteria decision-making (MCDM) methods have strongly been developed and have become increasingly popular (Ulutaş et al., 2020). Due to such strong development, a large number of multi-criteria decision-makings techniques have been introduced by scholars throughout the world (Stanujkic et al., 2019; 2017). So far, MCDM has successfully been applied in solving numerous decision-making problems in different areas. Significant progress in MCDM was made after Zadeh (1965) introduced fuzzy set theory, thus providing an efficient approach to solving a much larger number of real-world decision-making problems. An important characteristic of fuzzy sets is that they use the membership function to represent the degree of belonging to a set. Different shapes of membership functions are proposed, such as triangular, trapezoidal and bell-shaped so as to enable the efficient solving of different decision-making problems.

In order to enable solving more complex problems, some extensions of fuzzy set theory have been proposed, such as interval-valued fuzzy sets (Turksen, 1986), intuitionistic fuzzy sets (Atanassov, 1986), interval-valued intuitionistic fuzzy sets (Atanassov & Gargov, 1989), the bipolar fuzzy set (Lee, 2000), Pythagorean fuzzy sets (Yager, 2013) and so on.

In addition to more complex membership functions, these extensions also bring new mathematical operations on sets, as well as new membership functions. In bipolar fuzzy sets theory, Lee (2000) introduced the positive membership function $\mu_A^+(x)$ and the negative membership function $v_A^-(x)$, with the following conditions: $\mu_A^+(x) \in [0,1]$ and $v_A^-(x) \in [-1,0]$.

In intuitionistic fuzzy sets, Atanassov (1986) introduced the membership $T_A(x)$ and non-membership $F_A(x)$ functions, with the following conditions: $T_A(x), F_A(x) \in [0,1]$ and $T_A(x) + F_A(x) \le 1$. In intuitionistic fuzzy sets, indeterminacy $\pi_A(x)$ is $1 - T_A(x) - F_A(x)$ by default.

The condition $T_A(x) + F_A(x) \le 1$ can be a limitation for using intuitionistic fuzzy numbers. Therefore, in Pythagorean fuzzy sets, this condition is transformed into the following $T_A(x)^2 + F_A(x)^2 \le 1$ in order to facilitate their use in relation to intuitionistic fuzzy sets.

Smarandache (1999) proposed neutrosophic sets, as the generalization of fuzzy sets and their extensions. Neutrosophic sets have three independent membership functions, namely: the truth-membership function $T_A(x)$, the falsity-membership $F_A(x)$ function and the indeterminacy-membership $I_A(x)$ function. Smarandache (1999) and Wang et al. (2010) further proposed a single-valued neutrosophic set by modifying the conditions $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0,1]$ and

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$0 \leq T_A(x)+ I_A(x)+F_A(x) \leq 3$, which are more suitable for solving scientific and
engineering problems (Li et al., 2016).

Single-valued neutrosophic numbers are used for solving a number of
different problems, as a result of which many approaches have been proposed for
their ranking, such as ranking based on the score and accuracy functions (Sahin
2014), ranking based on the Euclidean and Hamming distances (Ye, 2015;
Pramanik et al., 2015), ranking based on different similarity measures (Ye, 2014).
There are a number of the similarity measures that can be used to rank single-
valued neutrosophic numbers. Therefore, an overview of some prominent
similarity measures is given below, and the rest of the paper is organized in the
following way: in Section 2, some basic concepts of neutrosophic sets and single-
valued neutrosophic numbers are presented; in Section 3, some distances and
similarity measures between n-dimensional vectors are considered, whereas in
Section 3.1, similarity measures between two single-valued neutrosophic numbers
are given; in Section 4, approaches for ranking of single-valued neutrosophic
numbers are presented; in Section 5, the evaluation of alternatives by using single
valued neutrosophic numberss is given; in Section 6, a group multi-criteria
decision-making approach based on the use of single-valued neutrosophic numbers
and similarity measures is presented, and in Section 7, an example is considered
with the aim of explaining the proposed methodology in detail. Section 8 is a
presentation of the analysis conducted and the conclusions of the paper are given in
the last section.

2. Some Basic Concepts of Neutrosophic Sets and Single-Valued
   Neutrosophic Numbers

Definition 1. Let X be the universe of discourse, with a generic element in
X denoted by x (Smarandache, 1999). The Neutrosophic Set (NS) A in X is an
object having the following form:

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| x \in X \right\},$$ (1)

with: $T_A : X \to]^-0,1^+[\; ; \qquad I_A : X \to]^-0,1^+[\; ; \qquad F_A : X \to]^-0,1^+[\qquad$ and

$^-0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+$, where: $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-
membership function, the indeterminacy-membership function, and the falsity-
membership function, respectively.

Definition 2. Let X be a nonempty set. The Single-Valued Neutrosophic
Set (SVNS) A in X is as follows (Wang et al., 2010):

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| x \in X \right\},$$ (2)

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with: $T_A : X \rightarrow [0,1]$; $I_A : X \rightarrow [0,1]$; $F_A : X \rightarrow [0,1]$ and

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$, where: $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the intermediacy-membership function, and the falsity-membership function, respectively, $T_A, I_A, F_A : X \rightarrow [0,1]$ and

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

The truth-membership, indeterminacy-membership, and falsity-membership functions are sometimes referred to as the truth, indeterminacy and falsity degrees, respectively.

Definition 3. A Single-Valued Neutrosophic Number (SVNN) $a = <t_a, i_a, f_a>$ is a special case of an SVNS on the set of real numbers $\Re$ where $t_a, i_a, f_a \in [0,1]$ and $0 \le t_a + i_a + f_a \le 3$ (Wang et al., 2010).

Definition 4. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs and $\lambda > 0$. The basic operations over the two SVNNs are as follows (Smarandache, 1999):

$$x_1 + x_2 = <t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2>. \tag{3}$$

$$x_1 \cdot x_2 = <t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2>. \tag{4}$$

$$\lambda x_1 = <1 - (1 - t_1)^\lambda, i_1^\lambda, f_1^\lambda>. \tag{5}$$

$$x_1^\lambda = <t_1^\lambda, i_1^\lambda, 1 - (1 - f_1)^\lambda>. \tag{6}$$

Definition 5. Let $x = <t_i, i_i, f_i>$ be an SVNN. The score function s_x of x is as follows (Sahin, 2014):

$$s_i = (1 + t_i - 2i_i - f_i)/2, \tag{7}$$

where $s_i \in [-1, 1]$.

Definition 6. Let $a_j = <t_j, i_j, f_j>$ be a collection of SVNSs and $W = (w_1, w_2, ..., w_n)^T$ be an associated weighting vector. The Single-Valued Neutrosophic Weighted Average (SVNWA) operator of a_j is as follows (Sahin, 2014):

$$SVNWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_j = \left(1 - \prod_{j=1}^{n}(1 - t_j)^{w_j}, \prod_{j=1}^{n}(i_j)^{w_j}, \prod_{j=1}^{n}(f_j)^{w_j} \right), \tag{8}$$

where: w_j is the element j of the weighting vector, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 7. Let $a_j = <t_j, i_j, f_j>$ be a collection of SVNSs and $W = (w_1, w_2, ..., w_n)^T$ be an associated weighting vector. Then, the Single-Valued Neutrosophic Weighted Geometric (SVNWG) operator of a_j is as follows (Sahin, 2014):

$$SVNWG(a_1, a_2, ..., a_n) = \prod_{j=1}^{n}(a_j)^{w_j} = \left(\prod_{j=1}^{n}(t_j)^{w_j}, 1 - \prod_{j=1}^{n}(1 - i_j)^{w_j}, 1 - \prod_{j=1}^{n}(1 - f_j)^{w_j} \right), \qquad (9)$$

where: w_j is the element j of the weighting vector, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$

3. Some Distances and Similarity Measures Between n-Dimensional Vectors

In this section, the definitions of the two distances and the four similarity measures between two n-dimensional vectors are presented.

Definition 8. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$.

The Hamming distance between X and Y is as follows:

$$h_{(X,Y)} = \sum_{i=1}^{n} |x_i - y_i|. \qquad (10)$$

Definition 9. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$. The Euclidean distance between X and Y is as follows:

$$e_{(X,Y)} = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}. \qquad (11)$$

Definition 10. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$ (Dice, 1945). The Dice similarity measure of the two vectors X and Y is as follows:

$$d_{(X,Y)} = \frac{2X \cdot Y}{\|X\|_2^2 + \|Y\|_2^2} = \frac{2\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2}. \qquad (12)$$

Definition 11. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$. The Jaccard similarity measure of the two vectors X and Y is as follows:

$$j_{(X,Y)} = \frac{X \cdot Y}{\|X\|_2^2 + \|Y\|_2^2 - X \cdot Y} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2 + \sum_{i=1}^{n} x_i y_i}. \qquad (13)$$

Definition 12. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$ (Salton & McGill, 1983). The cosine similarity measure of the two vectors X and Y is as follows:

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$$c_{(X,Y)} = \frac{X \cdot Y}{\|X\|_2 \|Y\|_2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2} \sqrt{\sum_{i=1}^{n} y_i^2}} .$$
(14)

Definition 13. Let $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ be two n-dimensional vectors, and $x_i, y_i \geq 0$ (Ye & Zhang, 2014). The similarity measure based on the minimum and maximum operators of the two vectors X and Y is as follows:

$$m_{(X,Y)} = \sum_{i=1}^{n} \frac{\min(x_i, y_i)}{\max(x_i, y_i)} .$$
(15)

3.1. The Similarity Measures of Two SVNNs

In this subsection, the similarity measures of two SVNNs are presented. In addition, the similarity measures between an SVNN and an ideal point, defined as <1, 0, 0>, are also considered.

Definition 14. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs. The similarity measure based on the Hamming distance of the two SVNNs is as follows:

$$h_{(x_1,x_2)} = 1 - \frac{1}{3}\left(|t_1 - t_2| + |i_1 - i_2| + |f_1 - f_2| \right).$$
(16)

Definition 15. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs. The similarity measure based on the Euclidean distance of the two SVNNs is as follows:

$$e_{(x_1,x_2)} = 1 - \left(\frac{1}{3}(|t_1 - t_2|^2 + |i_1 - i_2|^2 + |f_1 - f_2|^2) \right)^{\frac{1}{2}}.$$
(17)

Definition 16. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs (Ye, 2014). The Dice similarity measure of the two SVNNs is as follows:

$$d_{(x_1,x_2)} = \frac{2(t_1 t_2 + i_1 i_2 + f_1 f_2)}{(t_1^2 + i_1^2 + f_1^2) + (t_2^2 + i_2^2 + f_2^2)} .$$
(18)

Definition 17. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs (Ye, 2014). The Jaccard similarity measure of the two SVNNs is as follows:

$$j_{(x_1,x_2)} = \frac{t_1 t_2 + i_1 i_2 + f_1 f_2}{(t_1^2 + i_1^2 + f_1^2) + (t_2^2 + i_2^2 + f_2^2) + (t_1 t_2 + i_1 i_2 + f_1 f_2)} .$$
(19)

Definition 18. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs (Ye, 2014). The cosine similarity measure of the two SVNNs is as follows:

$$c_{(x_1,x_2)} = \frac{t_1 t_2 + i_1 i_2 + f_1 f_2}{\sqrt{t_1^2 + i_1^2 + f_1^2} \cdot \sqrt{t_2^2 + i_2^2 + f_2^2}} \, . \tag{20}$$

Definition 19. Let $x_1 = <t_1, i_1, f_1>$ and $x_2 = <t_2, i_2, f_2>$ be two SVNNs (Ye & Zhang, 2014). The similarity measure based on the minimum and maximum operators of the two SVNNs is as follows:

$$m_{(x_1,x_2)} = \frac{1}{3}\left(\frac{\min(t_1,t_2)}{\max(t_1,t_2)} + \frac{\min(i_1,i_2)}{\max(i_1,i_2)} + \frac{\min(f_1,f_2)}{\max(f_1,f_2)} \right). \tag{21}$$

4. The Ranking of Single-Valued Neutrosophic Numbers

There are several approaches to the comparison and ranking of SVNNs, such as: the ranking based on the use of the score function; the ranking based on distances from the ideal and anti-ideal points; and the ranking based on the use of similarity measures.

4.1. The ranking based on the use of the score function

The approach based on the use of the score function is commonly used. In this approach, the overall single-valued neutrosophic utility (OSVNU) of each alternative is first calculated by using Eq. (8), or Eq. (9), and then the score of each overall utility is determined by using Eq. (7). The alternative with a higher score is the most acceptable.Sahin (2014) and Mondal & Pramanik (2014) can be mentioned as some examples of the ranking of alternatives based on the use of the score function.

4.2. The ranking based on distances from the ideal and anti-ideal points

The ranking of alternatives based on their distances to the ideal point and the anti-ideal point is mainly used in the neutrosophic extensions of the TOPSIS method. The ranking of alternatives in such approaches is based on the application of the following equation:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \, , \tag{22}$$

where: c_i denotes the relative closeness to the ideal point of the alternative i, d_i^- and d_i^+ denote the distance of the alternative i to the anti-ideal point and the ideal point, respectively.

The alternative with the highest value of relative closeness is the most acceptable.

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Ye (2015), Pramanik et al. (2015) and Pouresmaeil et al. (2017) used the neutrosophic extensions of the TOPSIS method to solve different decision-making problems, such as the selection of the best investment alternative, the recruitment of an assistant teacher, and so forth. In these extensions of the use of both, the Euclidean and the Hamming distances are considered.

Biswas et al. (2016) used the neutrosophic extension of the TOPSIS method to select the most suitable tablet. In this approach, the Euclidean distance between two SVNNs was used.

4.3. The ranking based on the use of similarity measures

This approach is based on the similarity between the single-valued neutrosophic overall utility of an alternative and the single-valued neutrosophic ideal point. In a fashion similar to the case of the ranking based on the score function, the overall single-valued neutrosophic utility of each alternative is, first, calculated by using Eq. (8) or Eq. (9). Then, in the next step, the similarity measure between overall single-valued neutrosophic utility and the ideal point should be calculated. The alternative with the highest value of the similarity measure is the most acceptable.

The *single-valued neutrosophic* ideal point $r^* = <t^*, i^*, f^*>$ can be calculated as follows:

$$r^* = \begin{cases} \left\langle \max_i t_i, \min_i i_i, \min_i f_i \right\rangle, & j \in \Omega_{\max} \\ \left\langle \min_i t_i, \min_i i_i, \max_i f_i \right\rangle, & j \in \Omega_{\min} \end{cases}, \qquad (23)$$

where $\Omega_{\max}$ and $\Omega_{\min}$ denote a set of beneficial and nonbeneficial criteria, respectively.

In cases when all criteria are beneficial, Eq. (24) is applied as follows:

$$r^* = \left\langle \max_i t_{ij}, \min_i i_{ij}, \min_i f_{ij} \right\rangle, \qquad (24)$$

In neutrosophy, higher values for the truth degree and lower values for the falsity and indeterminacy degrees are preferable. Therefore, for the sake of simplicity, r_j^* could be determined as follows:

$$r^* = \left\langle 1, 0, 0 \right\rangle. \qquad (25)$$

In the case of using Eq. (25) instead of Eq. (24), the previously defined Eqs. (16)-(21) have the following forms:

Definition 20. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The similarity measure based on the Hamming distance of the SVNN x and the ideal point is as follows:

$$h_{(x)} = 1 - \frac{1}{3}\left(1 - t_x + i_x + f_x\right).$$

(26)

Definition 21. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The similarity measure based on the Euclidean distance between the SVNN x and the ideal point is as follows:

$$e_{(x)} = 1 - \left(\frac{1}{3}[(1 - t_x)^2 + i_x^2 + f_x^2]\right)^{\frac{1}{2}}.$$

(27)

Definition 22. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The Dice similarity measure between the SVNN x and the ideal point is as follows:

$$d_{(x)} = \frac{2t_x}{1 + t_x^2 + i_x^2 + f_x^2}.$$

(28)

Definition 23. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The Jaccard similarity measure between the SVNN x and the ideal point is as follows:

$$j_{(x)} = \frac{t_x}{1 + t_x^2 + i_x^2 + f_x^2 + t_x}.$$

(29)

Definition 24. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The cosine similarity measure between the SVNN x and the ideal point is as follows:

$$c_{(x)} = \frac{t_x}{\sqrt{t_x^2 + i_x^2 + f_x^2}}.$$

(30)

Definition 25. Let $x = <t, i, f>$ be an SVNN and let the ideal point be defined as <1, 0, 0>. The similarity measure based on the minimum and maximum operators between the SVNN x and the ideal point is as follows:

$$m_{(x)} = \frac{1}{3}t_x.$$

(31)

The following can be mentioned as some examples of the ranking of alternatives based on the use of similarity measures: Ye (2014) used three similarity measures, i.e. the Jaccard, Dice, and cosine similarity measures, to select

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the best investment alternative. Ye (2017) also used the cosine similarity measure for the fault diagnosis of a steam turbine. Chen et al. (2017) used similarity measures between two SVNNs to select the most appropriate construction project.

5. The Evaluation of Alternatives by Using SVNNs

The evaluation of alternatives in relation to evaluation criteria is a very significant step, especially so when interviewing the respondents who are unfamiliar with using neutrosophic numbers.

One possible way to perform evaluation more efficiently and more accurately is to form one statement sentence for each criterion, thus enabling the respondent to express his or her opinion regarding its accuracy and/or inaccuracy. For example, the following sentences can be used for the criteria delivery and price: *The supplier's delivery time is timely* and *the supplier provides fair prices*.

The values obtained in such a manner represent the truth degree and/or the falsity degree. In addition, the respondent may also express his or her uncertainty about the given answers. In such a case, the information obtained in that manner belongs to the indeterminacy degree. In some cases, the respondent may only use the truth degree, and then the falsity degree and the indeterminacy degree are set to zero.

Two linguistic scales were suggested in order to facilitate the evaluation of the alternatives. The first linguistic scale, shown in Table 1, can be used to express the level of the accuracy and inaccuracy of a statement sequence, while the second, shown in Table 2, can be used to express uncertainty about the given answers.

Table 1. The linguistic scale for expressing the level of accuracy and inaccuracy

Linguistic variable	$0 \div 1$	$0 \div 10$	$0 \div 100\%$
Extremely satisfied	≈ 1.00	≈ 10	$\approx 100\%$
Very satisfied	≈ 0.75	≈ 7.5	$\approx 75\%$
Moderately satisfied	≈ 0.50	≈ 5.0	$\approx 50\%$
Slightly satisfied	≈ 0.25	≈ 2.5	$\approx 25\%$
Dissatisfied	≈ 0.00	≈ 0.0	$\approx 0\%$

Table 2. The linguistic scale for expressing the level of uncertainty about the given answers

Linguistic variable	$0 \div 1$	$0 \div 10$	$0 \div 100\%$
Unconvinced	≈ 1.00	≈ 10	$\approx 100\ \%$
Poorly convinced	≈ 0.75	≈ 7.5	$\approx 75\ \%$
Moderately convinced	≈ 0.50	≈ 5.0	$\approx 50\ \%$
Convinced	≈ 0.25	≈ 2.5	$\approx 25\ \%$
Extremely convinced	≈ 0.00	≈ 0.0	$\approx 0\ \%$

The linguistic variables from both tables can be transformed into any of the proposed ranges, depending on the respondents' preferences, but the first interval $0 \div 1$ does not require normalization, whereas in the case of the second and the third ranges, the values should be transformed into the interval [0, 1]. The use of linguistic variables may make it easier for respondents to perform an evaluation, but in some ways, they can restrict the precise expression of their attitudes, which is why respondents should be informed that the given values are only approximate and that they may use any value from the interval [0, 1], or other scopes.

6. A Group Multi-Criteria Decision-Making Approach Based on Single-Valued Neutrosophic Numbers and Similarity Measures

The procedure for solving a multi-criteria decision-making problem that contains the m alternatives that are evaluated based on n criteria by K experts using SVNNs can precisely be expressed by the following algorithm:

Step 1. Define the goal of the evaluation and identify available alternatives.

Step 2. Form a group of experts who will perform the evaluation.

Step 3. Define a set of evaluation criteria and determine their significance, i.e. criteria weights.

Step 4. Form a questionnaire and define an evaluation scale, as discussed in Section 5.

Step 5. Evaluate the alternatives in relation to the selected criteria.

Step 6. Construct a group decision-making matrix by using Eq. (8) or Eq. (9).

Step 7. Calculate the single-valued neutrosophic overall utility of the alternatives by using Eq. (8) or Eq. (9).

Step 8. Determine the single-valued neutrosophic ideal point by using Eq. (24) or Eq. (25).

Step 9. Calculate a similarity measure for each alternative. Similarity measures can be calculated by applying any of Eqs (16)-(21) if the ideal point is determined by using Eq. (24), or by applying any of Eqs (26)-(31) if the ideal point is determined by using Eq. (25).

Step 10. Rank the alternatives and/or select the best one. The alternative with a higher similarity measure is more preferable.

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7. A Numerical Illustration

In order to briefly demonstrate the usability of SVNNs for solving MCDM problems, an example of the supplier selection is presented in this section.

Allow us to assume that one company must consider engaging a new supplier. Therefore, a team of three experts is formed with the aim of selecting the most appropriate supplier from the four alternatives denoted as A_1-A_4 based on the following criteria: C1 – Delivery; C2 – Quality; C3 – Flexibility; C4 – Service; and C5 – Price.

The set of the given criteria is determined based on the paper by Chang et al. (2011), who performed an evaluation of the criteria in order to determine which of the criteria are the most influential in the case of the supplier selection. As a result, the five criteria used in our case are singled out as the most significant.

The ratings obtained from the three experts, after their transformation into SVNNs and corrections, are presented in Tables 3, 4 and 5.

Table 3. The ratings obtained from the first of the three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.8, 0.30, 0.10>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>	<0.5, 0.0, 0.5>
A_2	<0.7, 0.00, 0.20>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>
A_3	<0.5, 0.00, 0.20>	<0.5, 0.0, 0.5>	<0.6, 0.0, 0.4>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>
A_4	<0.5, 0.20, 0.20>	<0.5, 0.2, 0.5>	<0.6, 0.2, 0.4>	<0.6, 0.2, 0.4>	<0.7, 0.2, 0.3>

Table 4. The ratings obtained from the second of the three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.6, 0.00, 0.40>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.0>	<0.6, 0.0, 0.0>
A_2	<0.8, 0.00, 0.20>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>	<0.8, 0.0, 0.2>	<0.6, 0.0, 0.0>
A_3	<0.7, 0.00, 0.30>	<0.8, 0.0, 0.2>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>
A_4	<0.7, 0.20, 0.30>	<0.8, 0.2, 0.2>	<0.7, 0.2, 0.3>	<0.6, 0.2, 0.4>	<0.7, 0.2, 0.3>

Table 5. The ratings obtained from the third of the three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.8, 0.50, 0.20>	<0.6, 0.5, 0.4>	<0.5, 0.5, 0.5>	<0.6, 0.5, 0.4>	<0.8, 0.5, 0.2>
A_2	<0.6, 0.20, 0.10>	<0.6, 0.2, 0.4>	<0.8, 0.2, 0.2>	<0.5, 0.2, 0.5>	<0.7, 0.2, 0.3>
A_3	<0.6, 0.10, 0.40>	<0.7, 0.1, 0.3>	<0.6, 0.1, 0.2>	<0.6, 0.1, 0.4>	<0.5, 0.1, 0.5>
A_4	<0.6, 0.20, 0.40>	<0.7, 0.2, 0.3>	<0.6, 0.2, 0.2>	<0.6, 0.2, 0.4>	<0.5, 0.2, 0.5>

As shown in Step 5 of the previously presented procedure for solving multi-criteria decision-making problems, the group decision-making matrix shown in Table 6 was constructed by using Eq. (8). In this calculation, all the three experts had the same significance, i.e. $w_j = 0.333$; $j = 1, 2$ and 3.

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Table 6. The group decision-making matrix

	C_1	C_2	C_3	C_4	C_5
A_1	<0.75, 0.0, 0.24>	<0.67, 0.0, 0.33>	<0.57, 0.0, 0.44>	<0.61, 0.0, 0.25>	<0.66, 0.0, 0.26>
A_2	<0.71, 0.0, 0.17>	<0.68, 0.0, 0.34>	<0.77, 0.0, 0.23>	<0.73, 0.0, 0.32>	<0.71, 0.0, 0.18>
A_3	<0.61, 0.0, 0.30>	<0.69, 0.0, 0.35>	<0.64, 0.0, 0.30>	<0.60, 0.0, 0.40>	<0.64, 0.0, 0.37>
A_4	<0.75, 0.0, 0.24>	<0.67, 0.0, 0.33>	<0.57, 0.0, 0.44>	<0.61, 0.0, 0.25>	<0.66, 0.0, 0.26>

In the next step, Step 6 of the above-proposed procedure, the overall ratings were calculated by using Eq. (7) and the following weights $w_j = \{0.18, 0.21, 0.20, 0.18, 0.23\}$. The overall ratings are shown in Table 7.

Table 7. The overall single-valued neutrosophic utility of the alternatives

	Overall utility
A_1	<0.65, 0.00, 0.30>
A_2	<0.72, 0.00, 0.24>
A_3	<0.64, 0.00, 0.34>
A_4	<0.65, 0.00, 0.30>

Finally, the ranking results obtained by using all the approaches considered in Section 3 and the ideal point $r^*=$<0.72, 0.00, 0.24>, calculated by using Eq. (24), are demonstrated in Table 8.

Table 8. The ranking results obtained by using the considered approaches

	I		II		III		IV		V		VI		VII	
	$h_{(i)}$		$e_{(i)}$		$d_{(i)}$		$j_{(i)}$		$c_{(i)}$		$mm_{(i)}$		$s_{(i)}$	
A_1	0.89	3	0.79	3	0.96	3	0.325	3	0.963	3	0.56	3	0.46	3
A_2	0.99	1	0.96	1	1.00	1	0.333	1	0.999	1	0.67	1	0.67	1
A_3	0.94	2	0.86	2	0.98	2	0.330	2	0.984	2	0.52	4	0.61	2
A_4	0.88	4	0.79	4	0.96	4	0.324	4	0.960	4	0.58	2	0.45	4

As can be seen from Table 8, all the considered approaches provide almost the same ranking orders of the evaluated alternatives. One discrepancy occurs when using the similarity measure based on the minimum and maximum operators of the two SVNNs.

In Column I of Table 8, the ranking results obtained by using the score function calculated by using Eq. (7) are shown. As can be seen, the ranking orders obtained by using the score function and the considered similarity measures are the same, except for the ranking order obtained by using the similarity measure based on the minimum and maximum operators of the two SVNNs. The obtained ranking orders of the alternatives are shown in Table 9.

Table 9. The ranking orders of the alternatives obtained by using the considered approaches

$h_{(i)}$	$e_{(i)}$	$d_{(i)}$	$j_{(i)}$	$c_{(i)}$	$mm_{(i)}$	$s_{(i)}$

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A_1	3	3	3	3	3	3	3
A_2	1	1	1	1	1	1	1
A_3	2	2	2	2	2	4	2
A_4	4	4	4	4	4	2	4

The similar ranking results are obtained in the case when the ideal point $r^*=<1.00, 0.00, 0.00>$ was used. The ranking results obtained in that way are shown in Table 10.

Table 10. The ranking results obtained by using the considered approaches

	I		II		III		IV		V		VI		VII	
	$h_{(i)}$		$e_{(i)}$		$d_{(i)}$		$j_{(i)}$		$c_{(i)}$		$mm_{(i)}$		$s_{(i)}$	
A_1	0.71	3	0.49	3	0.84	3	0.29	3	0.87	3	0.22	2	0.46	3
A_2	0.80	1	0.63	1	0.91	1	0.31	1	0.95	1	0.24	1	0.67	1
A_3	0.75	2	0.50	2	0.84	2	0.29	2	0.88	2	0.21	3	0.61	2
A_4	0.70	4	0.46	4	0.82	4	0.29	4	0.85	4	0.21	3	0.45	4

The obtained ranking orders are the same as those in the previous case, and the only discrepancy occurs when using the similarity measure based on the minimum and maximum operators of the two SVNNs.

8. The Analysis

To determine whether all the considered similarity measures generate the same or similar ranking orders of the alternatives, one theoretical case of the ranking in which the overall utility of the alternatives is represented by certain characteristic SVNNs is considered below, as is shown in Table 11.

Table 11. The overall utility of the alternatives

A_1	<1.00, 0.00, 0.00>
A_2	<1.00, 0.00, 1.00>
A_3	<1.00, 1.00, 0.00>
A_4	<1.00, 1.00, 1.00>
A_5	<0.00, 0.00, 0.00>
A_6	<0.00, 0.00, 1.00>
A_7	<0.00, 1.00, 0.00>
A_8	<0.00, 1.00, 1.00>
r^*	<1.00, 0.00, 0.00>

The reference point determined by using Eq. (24) is also shown in Table 11. The calculation details are presented in Table 12.

Table 12. The ranking results obtained by using the considered approaches

	I		II		III		IV		V		VI		VII	
	$h_{(i)}$		$e_{(i)}$		$d_{(i)}$		$j_{(i)}$		$c_{(i)}$		$mm_{(i)}$		$s_{(i)}$	
A_1	1.00	1	1.00	1	1.00	1	0.33	1	1.00	1	0.33	1	1.00	1
A_2	0.67	2	0.42	2	0.67	2	0.25	2	0.71	2	0.33	1	0.50	2
A_3	0.67	2	0.42	2	0.67	2	0.25	2	0.71	2	0.33	1	0.00	4
A_4	0.33	5	0.18	5	0.50	4	0.20	4	0.58	4	0.33	1	-0.50	6
A_5	0.67	2	0.42	2	0.00	5	0.00	5	0.00	5	0.00	5	0.50	2
A_6	0.33	5	0.18	5	0.00	5	0.00	5	0.00	5	0.00	5	0.00	4
A_7	0.33	5	0.18	5	0.00	5	0.00	5	0.00	5	0.00	5	-0.50	6
A_8	0.00	8	0.00	8	0.00	5	0.00	5	0.00	5	0.00	5	-1.00	8

In order to make a comparison, the value for the alternative A_5 obtained by using the cosine similarity measure is set to zero, because this similarity measure cannot be used when all the three membership functions are equal to zero. The obtained ranking results are shown in Table 13 and in Figure 1.

Table 13. The ranking orders of the alternatives obtained by using the considered approaches

	$h_{(i)}$	$e_{(i)}$	$d_{(i)}$	$j_{(i)}$	$c_{(i)}$	$mm_{(i)}$	$s_{(i)}$
A_1	1	1	1	1	1	1	1
A_2	2	2	2	2	2	1	2
A_3	2	2	2	2	2	1	4
A_4	5	5	4	4	4	1	6
A_5	2	2	5	5	5	5	2
A_6	5	5	5	5	5	5	4
A_7	5	5	5	5	5	5	6
A_8	8	8	5	5	5	5	8
max	8	8	5	5	5	5	8

Table 13 shows that the alternative A_1 is the best-ranked and the alternative A_8 is the worst-ranked in all the approaches. However, the ranking order of the other alternatives is slightly different.

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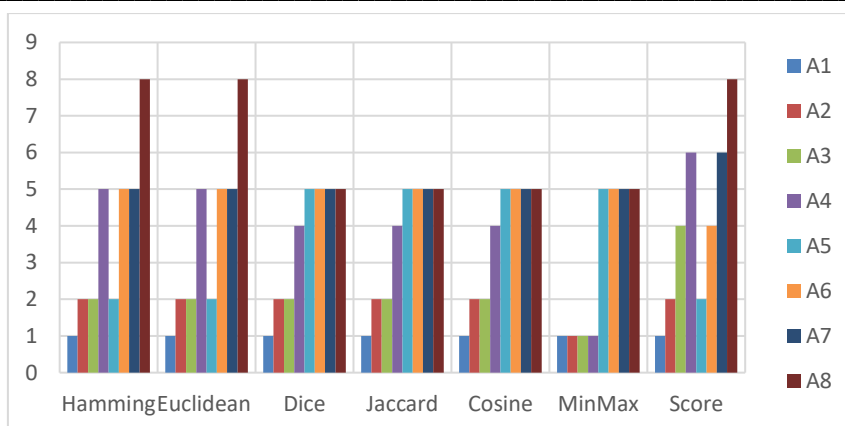Edmundas Kazimieras Zavadskas, Ieva Meidutė-Kavaliauskienė

Figure 1. The ranking differences as a result of applying different approaches

The largest discrepancy can be observed in the ranking obtained by using the similarity measure based on the minimum and maximum operators, whereas the highest similarity can be observed in the results obtained by using the Dice, Jaccard and Cosine measures. A high similarity can also be observed in the results obtained by applying the similarity measures based on the Hamming and Euclidean distances.

However, it should be noted that, in this example, the overall significance of the considered alternatives varied from ideally good to ideally bad. In real cases, it can be expected that differences will be less noticeable.

Conclusions

Neutrosophic sets extend the concept proposed in fuzzy and intuitionistic fuzzy sets. These sets enable us to use the three independent membership functions: the truth function, the intermediacy function and the falsity membership function. Therefore, neutrosophic sets can be very useful for solving complex decision-making problems and/or problems related to predictions and uncertainty.

In multi-criteria decision-making, neutrosophic sets enable the evaluation based on the use of a smaller number of more complex evaluation criteria. Despite their complexity, by using appropriate questionnaires, they can be used for an efficient evaluation of alternatives in relation to a set of evaluation criteria.

As a result of applying neutrosophic sets in solving numerous decision-making problems, a few approaches to the ranking of neutrosophic numbers based on similarity measures, as well as a few other, are proposed. Several prominent and less prominent similarity measures, such as the Dice, Jaccard and cosine similarity measures, measure similarity based on the minimum and maximum operators, and the similarity measures derived from the Hamming and Euclidean distances are presented and their use is considered in the case of the two numerical illustrations.

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One additional analysis was also carried out so as to determine whether similarity measures always generate the same or similar ranking orders of alternatives.

The obtained results indicate a significant similarity between the results obtained by using certain similarity measures, simultaneously also indicating that in certain cases there are evident differences.

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